Swing Set Dynamics

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Abstract

This paper models a swing set and tries to account for whether the exertion of one's legs or one's arms in isolation has more of an impact. The model seems to indicate that one's arms, or the position of one's torso has more of an impact, which contradicts the author's hypothesis. The conclusion explains how this unintuitive result might be sustained or rejected.

1 Introduction

The system this paper will model is a child's swing. As a child I recall wondering, was it possible to swing high enough that one would hit the beam overhead? Could one ever wrap around the top beam? Experience suggests this does not happen, but there are other questions one can investigate. How much do the extending of one's legs versus one's arms account for the motion of the swing? That is one of the questions this paper will investigate.

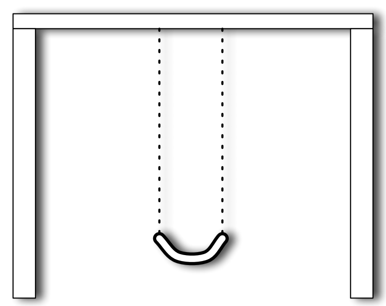

Figure 1: Diagram of typical physical swing

2 Physical System

The physical system modeled is a swing set that one would typically see on the playground, i.e., a frame supports a beam about ten feet or so high. The beam holds two chains of equal length hooked at the bottom to a seat. Figure 1 shows such a physical system from the front.

3 Model

This system is modeled in two dimensions. This does neglect some physical features like twisting or lateral motion (e.g., motion toward either side of the frame), but those features are not being investigated by this paper. Figure 2 shows a diagram of the 2D model and its generalized coordinates.

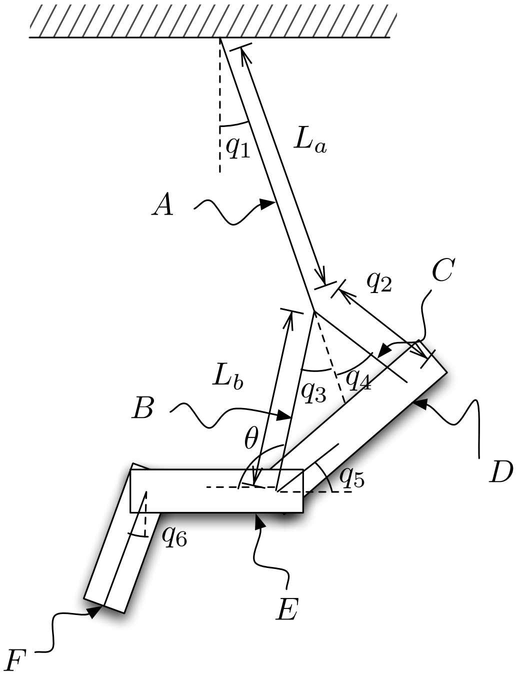

Figure 2: Model of swing showing generalized coordinates and body labels

The generalized coordinates are q_1 through q_6. L_a, L_b, and θ are constants. The boxes in Figure 2 labeled D, E, and F represent the body of the child, or actor, in the swing. The chain is represented by A and B. The system has holonomic constraints such that only four of the six generalized coordinates are independent. I choose to have q_2 and q_4 be dependent. Other holonomic constraints can be imposed so that the actor is not allowed to assume unnatural body positions, (e.g., q_6, the angle between body E and F, may be restricted to the range $[-\frac{\pi}{2}, \frac{\pi}{2}]$). The actor is expected to apply torque to q_6, and shrink or extend the agent's arms effectively changing the length of q_2.

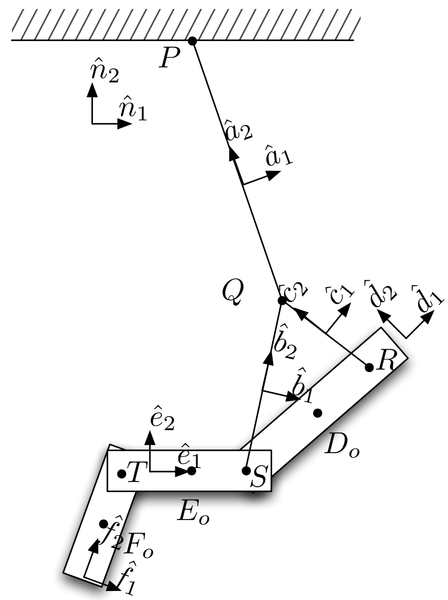

Figure 3: Model of swing with points and axes shown

3.1 External Forces

Gravity acts on each of the bodies D, E, and F. Friction acts at the points P and Q. Fiction constants Fr are incorporated into the model as shown in (1) and (2).

$$\vec{T}^{N/A} = -Fr_1 \ {}^{N}\vec{\omega}^{A} \tag{1}$$

$$\vec{T}^{A/B} = -Fr_2 \ {}^{A}\vec{\omega}^{B} \tag{2}$$

3.2 Controller

$$T_p \approx 2\pi\sqrt{\frac{L}{g}} \tag{3}$$

$$L = L_a + L_b \tag{4}$$

Since this model includes an agent that exerts forces and torques on the system, a controller is necessary. To a good approximation a pendulum's period is independent of its amplitude as shown in Equation 3; using this invariance, the controller for the torque applied to the legs (5) and the extension of the arms (8) can be a sinusoidal with the same period. Torques, forces, and other physical quantities will be denoted using Kane's notation[2].

$$\vec{T}^{F/E} = A \ sin(\frac{t2\pi}{T_p}) \ \vec{n}_3 \tag{5}$$

$$\vec{F}^{R} = -K(q_2 - L_{q_2}) \ \vec{c}_2 \tag{6}$$

$$\vec{F}^{Q} = K(q_2 - L_{q_2}) \ \vec{c}_2 \tag{7}$$

$$L_{q_2} = a \ sin(\frac{t2\pi}{T_p}) + b \tag{8}$$

A, K, a, and b are all constant parameters and g is the local gravitational acceleration.

Using this controller, the agent ought to mimic the behavior typically shown by a person on a swing: legs and arms extended moving forward, legs and arms retracted moving backward.

3.3 Parameters

Not all parameters are labeled in Figure 2 and Figure 3. There are lengths L_d, L_e, L_f for body D, E, and F respectively. Widths for each body W_d, W_e, and W_f are also required to calculate the moments of inertia.

3.4 Selecting Parameters

Selecting values for each parameter can be challenging. I elected to select them in this way. For the parameters relating to the body segments length, width, and mass, I used an anthropometry reference [1]. For the parameters relating to the swing set, I used recorded lengths from a local swing set. However, that still leaves other constants like the friction coefficients. One could eye ball the simulation and see whether it behaves physically realistically. I wanted to be able to compare with some data obtained from a physical system.

I recorded data using an accelerometer (available on the iPhone) on a swing under the following cases:

1. Once the swing is going, no leg and arm extensions.

2. Leg extensions but no arm extensions.

3. Arm extensions but no leg extensions.

4. Arm and leg extensions.

The accelerometer data allowed me to compare a real system with data produced by the simulation. For example, Figure 4 shows the magnitude of acceleration recorded from a real swing with the device placed in my lap. Figure 5 shows the magnitude of acceleration at point E_o from the simulation. Both were case 4, arm and leg extensions.

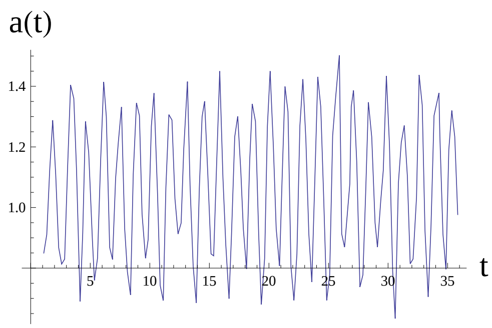

Figure 4: Magnitude of acceleration (units g) over time in case 4 recorded on a real swing

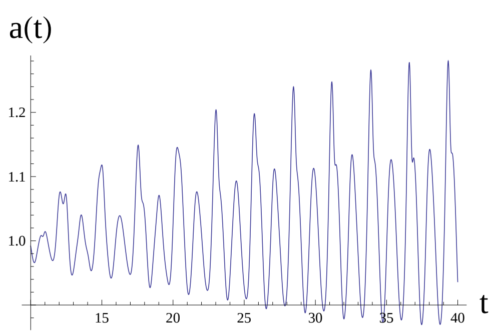

Figure 5: Magnitude of acceleration (units g) over time in case 4 obtained from simulation at point E_o

4 Hypothesis

There are a handful of questions that I would like to address.

1. How effective is using only one's legs?
2. How effective is using only one's arms?
3. How effective is using one's legs and arms?

My operational definition for effective is the max height one can achieve using a method in a limited time; the higher, the more effective, and height is proportionate to q_1. My hypothesis is that using one's arms and legs will demonstrate an increase in efficacy that is better than the sum of each in isolation. I also expect that only using one's legs to do better than only using one's arms.

5 Results

AUTOLEV[3] was used to model the physics of the swing set. *Mathematica* was employed to run the simulations. A graphical representation of the simulation is shown in Figure 6. Figure 7 Case 1 shows that friction is slowly decreasing the amplitude of the oscillation, which is what one would expect. Figure 8 Case 2 shows that leg extensions in this model do not appear very effective. Figure 9 Case 3 shows that arm extensions in this model are effective, more effective than the legs. Figure 10 Case 4 shows that arm and leg extensions are effective in combination, seemingly more effective than either alone. Let us inspect more closely.

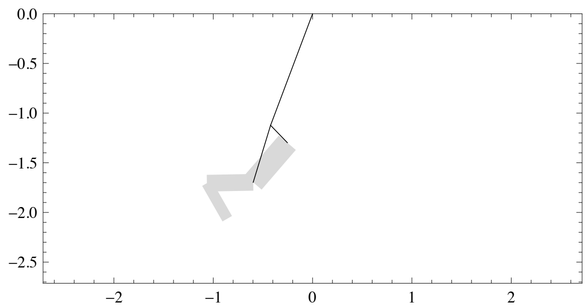

Figure 6: View of simulation

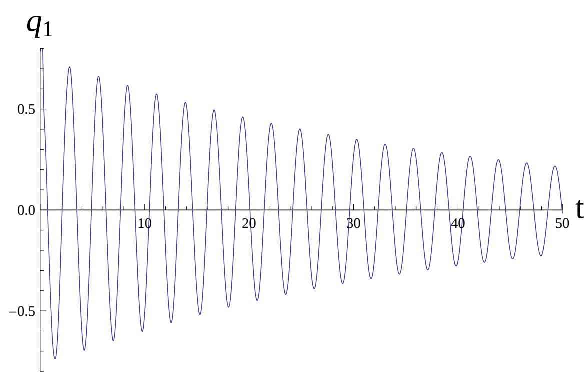

Figure 7: Case 1: Swing is moving but no leg or arm extensions

Figure 11 shows Case 4 in red, and Case 2 plus Case 3 (blue) added together. Case 4 is more effective than Case 2 and Case 3 separately. However, examining Figure 12 it seems that the difference between the two becomes more negligible as the amplitude increases.

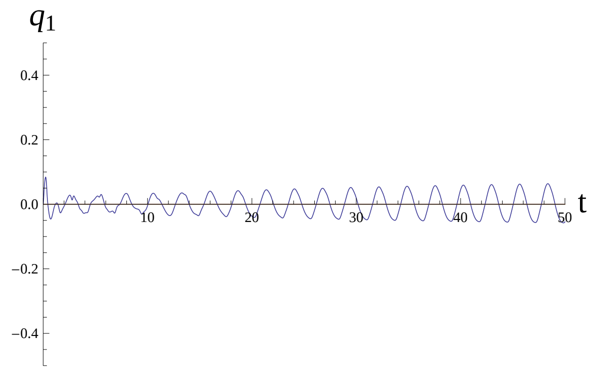

Figure 8: Case 2: Leg extensions but no arm extensions

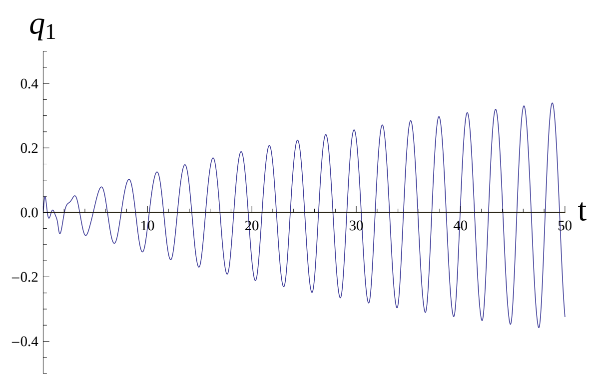

Figure 9: Case 3: Arm extensions but no leg extensions

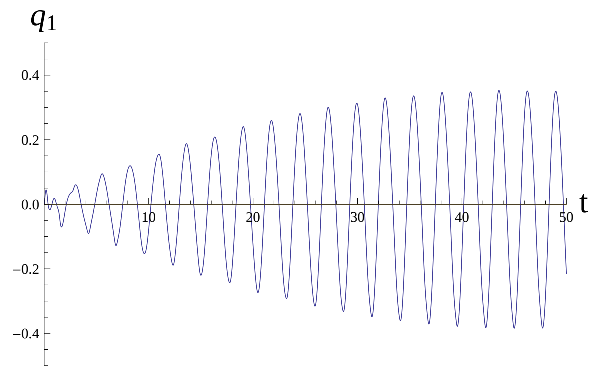

Figure 10: Case 4: Arm and leg extensions

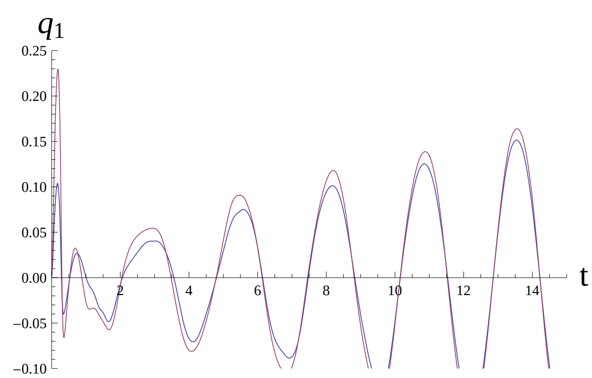

Figure 11: Close up of Case 4 (red) compared with Case 2 plus Case 3 (blue)

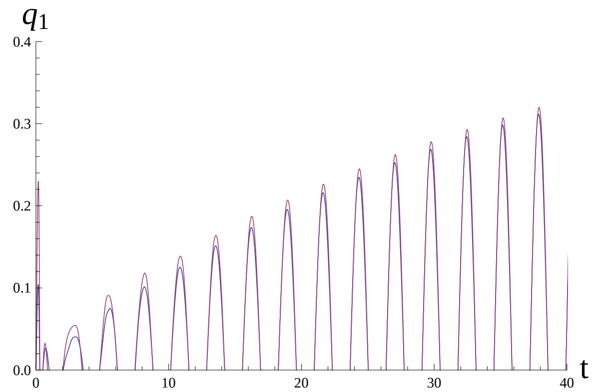

Figure 12: Case 4 (red) compared with Case 2 plus Case 3 (blue) shown further out

6 Conclusion

This paper modeled a physical system, successfully achieved a qualitative correspondence with accelerometer data, and simulated that system to try and answer a question about that physical system. The results for Case 2 run counter to my intuition. I expected leg extensions to be more effective than arm extensions. I am inclined to go back to the playground and see if these results are reproducible. More of the parameter space may need to be explored to say something definitively.

The hypothesis that using one's arms and legs in combination were more effective than either was affirmed but not by a significant amount. The effect of which seems to become negligible over time. It could be the case that the combination of extending one's arms and legs increases the speed at which one reaches a particular amplitude but not the maximum amplitude.

Despite my intuitions, Case 2 may demonstrate that when one starts swinging with no initial speed nor any angle with respect to the bar overhead ($q_1 = u_1 = 0$) , merely swinging one's legs may not be very effective. We may out of habit give ourselves a boost by pulling back and creating an angle such that the initial $q_1 \neq 0$. More time at a playground and more canvassing of the parameter space would help reveal whether the results for Case 2 shown in this paper are robust, a special case, or erroneous.

7 Acknowledgments

I want to thank Professor Hubbard for the introduction to Kane's method and taking the time to discuss this project. Accelerometer data acquired using the *xSensor* iPhone application.

References

[1] Harry G. Armstrong. *Anthropometry. and Mass Distribution for Human Analogues*, volume Volume I: Military Male Aviators. Anthropology Research Project, March 1988.

[2] Thomas R. Kane and David A. Levinson. *Dynamics, Theory and Applications*. McGraw Hill, 1985.

[3] Thomas R. Kane and David A. Levinson. *Dynamics online: theory and implementation with AUTOLEV*. Online Dynamics, Inc, 1996.